MAXIMUM LIKELIHOOD ESTIMATION OF CLUTTER SUBSPACE IN NON HOMOGENEOUS NOISE CONTEXT

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ABSTRACT

In the context of a disturbance composed of a Low Rank (LR) clutter plus a white Gaussian noise, the corresponding LR filters used to detect a target embedded in this disturbance needs less training vectors than classical methods to reach equivalent performance. Unlike the classical one which is based on covariance matrix of the noise, the LR filter is based on the clutter subspace projector. In this paper, we propose a new estimator of the clutter subspace projector for a disturbance composed of a LR Spherically Invariant Random Vectors (SIRV) plus a zero mean white Gaussian noise that does not require prior information on the SIRV's texture. Numerical simulations validate the introduced estimator, and its performance and robustness are tested on a Space Time Adaptive Processing (STAP) simulation.

***Index Terms*—** Covariance Matrix and Projector estimation, Maximum Likelihood Estimator, Low-Rank clutter, SIRV, STAP filter.

1. INTRODUCTION

In array processing, such as STAP Radar [1], the optimal filter in terms of Signal to Noise Ratio (SNR) is composed by the inverse of the Covariance Matrix (CM) of the noise and the steering vector. In practice, the CM of the noise is unknown and have to be primary estimated with a set of secondary data: K realizations of the noise without any signal of interest. The CM estimate is then used to process sub-optimal filtering.

The CM estimator typically used is the Sample Covariance Matrix (SCM), which is the Maximum Likelihood Estimator (MLE) of the CM in a Gaussian environment. In this case, $2m$ (m is the size of the data) secondary data are needed to ensure good performance of the sub-optimal filtering, i.e. a 3dB loss of the output SNR compared to optimal filtering [2]. When the the noise is composed of a low-rank (LR) clutter plus a white Gaussian noise, the corresponding sub-optimal filter is based on the projection of the received data onto the orthogonal subspace of the clutter subspace [3]. Estimating the clutter subspace projector requires $2r$ secondary data (r is the clutter rank, and often $r \ll m$) to reach equivalent performance to the previous scheme [4]. Classically, this projector estimate is derived from the Singular Value Decomposition (SVD) of the SCM. Nevertheless, the SCM is not adapted for a non-Gaussian noise such as heterogeneous clutter and developing filters/detectors on it may lead to poor performance. In this paper, the clutter noise is modeled as SIRVs, first introduced by [5], known for their good agreement to empirical data sets [6]. A SIRV process is a compound Gaussian mixture with a random power factor called the *texture*. The clutter subspace estimate may be then derived from the SVD of the Fixed-Point estimator (FPE), which is an approached MLE of the CM for SIRV noise [7, 8]. However, the FPE is not the MLE of the CM in the described context: LR-SIRV plus white Gaussian noise. Moreover, it requires $K > m$ secondary data to be computed, which does not allow to take full advantage of the LR assumption in the cases where $2r \ll m$.

In this paper, we propose to develop a direct estimator of the clutter subspace projector via MLE. This approach had been inspired by [9], where such an estimator had been given under several hypothesis: the CM of the low-rank clutter is assumed to have identical eigenvalues, and the Probability Density Function (PDF) of the texture is assumed known. In this paper, we propose to relax the second assumption, and therefore introduce a new estimator of the clutter subspace projector with no prior information on the texture PDF in the context of a LR SIRV clutter plus a white Gaussian noise. This new estimator is compared with the classical one based on SCM and those proposed in [9] to quantify the loss of performance when prior information on texture is not taken into account. The robustness of the proposed estimator is also studied on a realistic simulation of STAP Radar, where eigenvalues of the LR clutter are not identical.

The following convention is adopted: italic indicates a scalar quantity, lower case boldface indicates a vector quantity and upper case boldface a matrix. ^{H} denotes the transpose conjugate operator. $\mathscr{CN}(\mathbf{a}, \mathbf{R})$ is a complex Gaussian vector of statistical mean $\mathbf{a}$ and of covariance matrix $\mathbf{R}$. $\mathbf{I}_m$ is the $m \times m$ identity matrix. $\hat{d}$ is the ML estimate of the statistical parameter d. $\{w_i\}_{i=1,\ldots,n}$ denotes the set of n elements w_i with $i = 1, ..., n$ and whose writing is sometimes contracted in $\{w_i\}$.

2. STATISTICAL MODEL AND EXPRESSION OF THE LIKELIHOOD FUNCTION

We assume that K secondary data are available. The noise is modeled as a LR-SIRV process plus an additive zero-mean complex white Gaussian noise. A realization of a SIRV process is a Gaussian random vector with a random power factor called the *texture* τ_i. The texture is here considered as an unknown deterministic positive parameter. Therefore, each data $\mathbf{z}_i \in \mathbb{C}^m, i = 1, ..., K$ can be described by $\mathbf{z}_i \sim \mathscr{CN}(\mathbf{0}, \mathbf{R}_i)$, with

$$\mathbf{R}_i = \sigma^2 \mathbf{I}_m + \tau_i \sum_{k=1}^{r} c_k \mathbf{v}_k \mathbf{v}_k^H, \tag{1}$$

where $\sigma^2 \mathbf{I}_m$ represents the CM of the white Gaussian noise. The power of this noise σ^2 may be considered unitary since the data can be normalized by it (or its estimate), therefore $\sigma^2 = 1$. The CM of the clutter subspace is described by its rank r, its eigenvalues c_k and associated eigenvectors $\mathbf{v}_k$, $k = 1, ..., r$. The clutter rank r is assumed to be known. In classical STAP, r can be evaluated thanks to the Brennan rule [10]. Moreover, the CM of the clutter will be assumed to have a low-rank structure ($r \ll m$). We will also, as [9], assume that $c_k = 1$ for $k = 1, ..., r$. Thus, by denoting the clutter subspace projector $\mathbf{\Pi}_c = \sum_{k=1}^{r} \mathbf{v}_k \mathbf{v}_k^H$, the simplified CM is:

$$\mathbf{R}_i = \mathbf{I}_m + \tau_i \mathbf{\Pi}_c \tag{2}$$

The hypothesis of equals c_k is mainly imposed by tractability purpose. Nevertheless section 4.3 will show that the estimator derived from this model seems robust to an inequality of the c_k's. A possible interpretation of (2) is that we consider that the texture τ_i "absorbs" the factors c_k: The average power of the clutter is uniformly applied over its corresponding subspace.

In [9], the texture τ_i is treated as a random variable with a known PDF. This hypothesis of prior knowledge is restrictive in practical cases, where the texture is unknown and even its PDF is not usually available. Therefore, τ_i is considered in this paper as an unknown deterministic parameter. The likelihood of the data, conditioning to $\mathbf{\Pi}_c$ and $\{\tau_i\}_{i=1,...,K}$ is then:

$$f(\mathbf{z}_1, ..., \mathbf{z}_K | \mathbf{\Pi}_c, \{\tau_i\}) = \prod_{i=1}^{K} \frac{e^{-\mathbf{z}_i^H \mathbf{R}_i^{-1} \mathbf{z}_i}}{\pi^m |\mathbf{R}_i|} \tag{3}$$

Since $\mathbf{R_i}^{-1} = (\mathbf{I} - \mathbf{\Pi}_c) + \frac{1}{1+\tau_i} \mathbf{\Pi}_c$ and $|\mathbf{R}_i| = (\tau_i + 1)^r$, equation (3) becomes:

$$f(\mathbf{z}_1, ..., \mathbf{z}_K | \mathbf{\Pi}_c, \{\tau_i\}) = \prod_{i=1}^{K} \frac{e^{-\mathbf{z}_i^H (\mathbf{I}_n - \frac{\tau_i}{1+\tau_i} \mathbf{\Pi}_c) \mathbf{z}_i}}{\pi^m (\tau_i + 1)^r} \tag{4}$$

One can then express the log-likelihood function of the data set:

$$\ln(f(\mathbf{z}_1, ..., \mathbf{z}_K | \mathbf{\Pi}_c, \{\tau_i\})) = -\sum_{i=1}^{K} \mathbf{z}_i^H \mathbf{z}_i$$
$$+ \sum_{i=1}^{K} \frac{\tau_i}{1+\tau_i} \mathbf{z}_i^H \mathbf{\Pi}_c \mathbf{z}_i - Km \ln \pi - r \sum_{i=1}^{K} \ln(\tau_i + 1) \tag{5}$$

Let us now address the problem of the estimation of $\mathbf{\Pi}_c$. Since $\mathbf{\Pi}_c = \sum_{k=1}^{r} \mathbf{v}_k \mathbf{v}_k^H$, the problem is directly equivalent to the estimation of a basis of the clutter subspace $\{\mathbf{v}_k\}_{k=1,...,r}$.

3. ML ESTIMATOR OF THE CLUTTER SUBSPACE

3.1. Expression of the clutter subspace MLE

The estimation method goes as follows: the MLEs of the unknown textures τ_i, $i = 1, ..., K$, are firstly determined from (5), these parameters are then replaced by their MLEs expressions in (5) to obtain the generalized log-likelihood $\hat{f}$ and finally this functional (plus a normalization constraint function) is derived with respect to $\mathbf{v}_k$, $k = 1, ..., r$ which leads to the expression of the clutter subspace basis MLE.

Lemma 3.1 *The MLE under positivity constraint of τ_i, $i = 1, ..., K$ conditional to $\mathbf{\Pi}_c$, denoted $\hat{\tau}_i$, is:*

$$\hat{\tau}_i = \begin{cases} \frac{1}{r} \mathbf{z}_i^H \mathbf{\Pi}_c \mathbf{z}_i - 1 & \text{if } ||\mathbf{\Pi}_c \mathbf{z}_i||^2 > r \\ 0 & \text{else} \end{cases} \tag{6}$$

Proof By taking the derivative of the expression (5) with respect to (w.r.t.) τ_i, for a specific $i \in 1, ..., K$:

$$\frac{\partial \ln(f(\mathbf{z}_1, ..., \mathbf{z}_n | \tau_i))}{\partial \tau_i} = \frac{\mathbf{z}_i^H \mathbf{\Pi}_c \mathbf{z}_i}{(1+\tau_i)^2} - \frac{r}{\tau_i + 1} \tag{7}$$

This equation is canceled to identify $\hat{\tau}_i$, the MLE of τ_i. Nevertheless the texture is known to be a positive value. Since the likelihood is strictly decreasing after his maximum $\hat{\tau}_i$, the MLE under the positivity constraint is given by (6).

Proposition 3.2 *The ML basis of the clutter subspace is defined by the $\{\hat{\mathbf{v}}_k\}_{k=1,...,r}$ that are the r most important eigenvectors of the matrix $\hat{\mathbf{M}}(\mathbf{\Pi}_c)$:*

$$\hat{\mathbf{M}}(\mathbf{\Pi}_c) = \sum_{i=1}^{K} \frac{\hat{\tau}_i}{\hat{\tau}_i + 1} \mathbf{z}_i \mathbf{z}_i^H \tag{8}$$

Proof The τ_i's, are replaced in by their MLE expression $\hat{\tau}_i$, $i = 1, ..., K$ (5) in order to obtain the generalized log-likelihood $\hat{f}$:

$$\ln(\hat{f}(\mathbf{z}_1, ..., \mathbf{z}_n | \mathbf{v}_1, ..., \mathbf{v}_r)) = -\sum_{i=1}^{K} \mathbf{z}_i^H \mathbf{z}_i$$
$$+ \sum_{i=1}^{K} \mathbf{z}_i^H \mathbf{\Pi}_c \mathbf{z}_i - Km \ln \pi - Kr - r \sum_{i=1}^{K} \ln(\frac{1}{r} \mathbf{z}_i^H \mathbf{\Pi}_c \mathbf{z}_i) \tag{9}$$

The vectors $\mathbf{v}_k$, $k = 1,...,r$ must form a basis of the clutter subspace estimate. Thus, the maximization of $\hat{f}$ with respect to the $\mathbf{v}_k$'s must be done under a normalization constraint. Nevertheless, imposing an orthogonality constraint is not necessary since the solution will appear as eigenvectors of an unique matrix and therefore inherently orthogonal to each other. The functional g to maximize w.r.t the $\mathbf{v}_k$'s is:

$$\ln(g(\mathbf{z}_1,...,\mathbf{z}_K)) = -\sum_{i=1}^{K} \mathbf{z}_i^H \mathbf{z}_i + \sum_{i=1}^{K}\sum_{k=1}^{r} \mathbf{z}_i^H \mathbf{v}_k \mathbf{v}_k^H \mathbf{z}_i - Km\ln\pi - Kr - r\sum_{i=1}^{K} \ln\left(\frac{1}{r}\sum_{k=1}^{r} \mathbf{z}_i^H \mathbf{v}_k \mathbf{v}_k^H \mathbf{z}_i\right) + \sum_{k=1}^{r} \lambda_k (\mathbf{v}_k^H \mathbf{v}_k - 1) \quad (10)$$

where λ_k, $k = 1,...,r$ are Lagrange multipliers associated to the normalization constraint. The functional g is differentiated, then canceled w.r.t. $\mathbf{v}_j^H$ for a specific $j \in 1,...,r$ to obtain the expressions of the clutter subspace vectors estimators:

$$\frac{\partial g(\mathbf{z}_1,...,\mathbf{z}_K)}{\partial \mathbf{v}_j^H} = 0 \Leftrightarrow$$

$$\sum_{i=1}^{K} \mathbf{z}_i \mathbf{z}_i^H \mathbf{v}_j - r\sum_{i=1}^{K} \frac{\mathbf{z}_i \mathbf{z}_i^H \mathbf{v}_j}{\mathbf{z}_i^H \left(\sum_{k=1}^{r} \mathbf{v}_k \mathbf{v}_k^H\right) \mathbf{z}_i} = \lambda_j \mathbf{v}_j, \quad (11)$$

where the expression of the $\hat{\tau}_i$'s given from Lemma 3.1 is identified:

$$(11) \Leftrightarrow \left(\sum_{i=1}^{K} \frac{\hat{\tau}_i}{\hat{\tau}_i + 1} \mathbf{z}_i \mathbf{z}_i^H\right) \mathbf{v}_j = \lambda_j \mathbf{v}_j \quad (12)$$

Thus, the ML basis of the clutter subspace is defined by the $\{\hat{\mathbf{v}}_k\}_{k=1,...,r}$ that are the r most important eigenvectors of the matrix $\hat{\mathbf{M}}(\mathbf{\Pi}_c)$:

$$\hat{\mathbf{M}}(\mathbf{\Pi}_c) = \sum_{i=1}^{K} \frac{\hat{\tau}_i}{\hat{\tau}_i + 1} \mathbf{z}_i \mathbf{z}_i^H \quad (13)$$

One may notice that the matrix defining the $\{\hat{\mathbf{v}}_k\}$ is a SCM of the data scaled by a factor that give more weight to $\mathbf{z}_i$'s with a strong Clutter to Overall Noise Ratio ($CONR$). This factor is in fact the $CONR$ of the considered $\mathbf{z}_i$ MLE, which expression depends on the texture estimates. In other words, realizations that contain more power in the subspace of interest are given more significance in the estimation process.

3.2. Remark on computation

Proposition 3.2 gives an expression where the clutter subspace estimator is depending on this subspace itself. Indeed, $\mathbf{\Pi}_c$ is estimated using $\hat{\tau}_i$, which is estimated conditioning to $\mathbf{\Pi}_c$. The same problem appears in [9], and have been solved by using an Expectation-Maximization (EM) [11] approach which leads to a recursive algorithm. We propose in this paper an equivalent algorithm to approach the fixed point of the MLE solution of proposition 3.2. The process consists in estimating the $\hat{\tau}_i^{(n)}$'s with

$$\hat{\tau}_i^{(n)} = \begin{cases} \frac{1}{r}\mathbf{z}_i^H \hat{\mathbf{\Pi}}_c^{(n)} \mathbf{z}_i - 1 & \text{if } ||\hat{\mathbf{\Pi}}_c^{(n)} \mathbf{z}_i||^2 > r \\ 0 & \text{else} \end{cases} \quad (14)$$

then on picking out the r most important eigenvectors of the matrix:

$$\hat{\mathbf{M}}^{(n+1)}(\hat{\mathbf{\Pi}}_c^{(n)}) = \sum_{i=1}^{K} \frac{\hat{\tau}_i^{(n)}}{\hat{\tau}_i^{(n)} + 1} \mathbf{z}_i \mathbf{z}_i^H \quad (15)$$

to obtain the updated clutter subspace projector estimate $\hat{\mathbf{\Pi}}_c^{(n+1)}$. This algorithm is an alternate maximization of the function $\hat{f}$ (bounded), therefore its convergence is ensured. Nevertheless, it could reach local maximums of the function and should be carefully initialized. $\mathbf{\Pi}_c^{(0)}$ will be here given by the r most important eigenvectors of the SCM since it is, as discussed in [9], good initial guess. The convergence of this algorithm will also be illustrated by simulations in section 4.1.

4. SIMULATIONS RESULTS

This section deals with numerical simulations to illustrate the performance of the proposed estimator. Two criteria of performance are studied. The first one, called Power-Suppression [9], represents the average "accuracy" of the subspace estimation. The second one, the Signal to Interference plus Noise Ratio (SINR) [1] loss, is linked to classical STAP Radar filtering performance.

For comparison purposes, two clutter subspace MLEs presented in [9] are briefly recalled:

- Clutter subspace estimator with known texture :
  This correspond to the optimal estimation procedure but is not realizable in practice. It will be used as a theoretical benchmark. If the texture τ_i is known for each realization, the ML basis of the clutter subspace is defined by the $\{\hat{\mathbf{v}}_k\}_{k=1,...,r}$ that are the r most important eigenvectors of the matrix $\mathbf{M}$:

$$\mathbf{M} = \sum_{i=1}^{K} \frac{\tau_i}{\tau_i + 1} \mathbf{z}_i \mathbf{z}_i^H \quad (16)$$

- Clutter subspace estimator with known texture PDF :
  If the texture PDF, denoted f_τ, is known, the ML basis of the clutter subspace is defined by the $\{\hat{\mathbf{v}}_k\}_{k=1,...,r}$ that are the r most important eigenvectors of the matrix

$\mathbf{M}_f$:

$$\mathbf{M}_f = \sum_{i=1}^{K} \left[1 + \frac{h'\left(\sum_{k=1}^{r} \mathbf{z}_i^H \mathbf{v}_k \mathbf{v}_k^H \mathbf{z}_i\right)}{h\left(\sum_{k=1}^{r} \mathbf{z}_i^H \mathbf{v}_k \mathbf{v}_k^H \mathbf{z}_i\right)} \right] \mathbf{z}_i \mathbf{z}_i^H, \tag{17}$$

with

$$h(q) = \int_0^\infty \frac{\exp -q/(1+\tau_i)}{(1+\tau_i)^r} f_\tau(\tau_i) d\tau_i \tag{18}$$

4.1. Convergence of the algorithm

For these simulations, secondary data have been generated according to the LR clutter plus white Gaussian noise model specified in Eq. (2). The PDF associated to the texture is here a discrete law of the form:

$$f_\tau(y) = \sum_{n=1}^{N_Y} p_n \delta(y - a_n) \tag{19}$$

The values of the parameters of f_τ have been set to: $N_Y = 3$, $(a_1, a_2, a_3) = \alpha(0.1, 1, 100)$ and $(p_1, p_2, p_3) = (0.5, 0.4, 0.1)$. The factor α is a parameter used to set the Clutter to Noise Ratio (CNR).

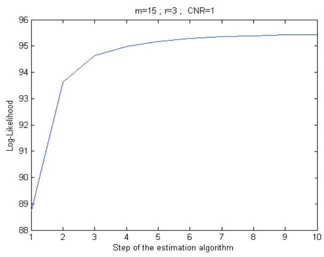

Fig. 1. Mean Log-Likelihood (on 1000 trials) in function of the algorithm step. $m = 15$, $r = 3$, $CNR = 0$dB

Figure 1 shows the convergence of the algorithm described in 3.2 to compute the proposed estimator. It presents the mean Log-Likelihood (on 1000 trials) versus the algorithm step.

4.2. Power Suppression

Theses simulations present a comparison between the proposed estimator, the classical one and those based on Eqs (16) and (17). Secondary data have been generated as in the previous section. The criterion used to compare the methods is the Power Suppression [9], namely:

$$\text{Power-Suppression} = 20\log_{10}(\sin(\phi)), \tag{20}$$

where ϕ is the maximum angle between the estimated subspace and the true one.

Figure 2 shows the Power-Suppression evolution with respect to K and CNR with 3 iterations for the iterative estimators. As it could be expected, the performance of our new estimator is better than the classical one based on SCM, especially for large K and low CNR. Indeed, if the CNR is high enough, the performance of the SCM estimator is equivalent to the others due to the small contribution of the white Gaussian noise relatively to the LR-SIRV process. The method of [9] performs better estimation than the proposed method but cannot be used in practice if the texture PDF is unknown.

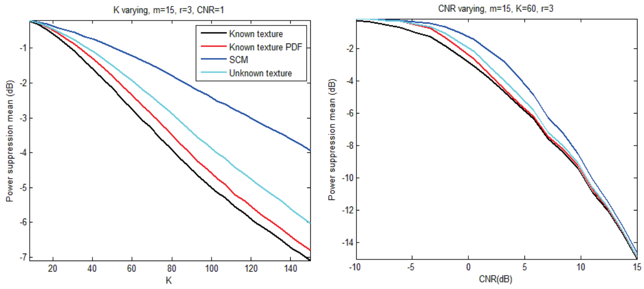

Fig. 2. Power-Suppression mean on 10000 iterations for K varying with $CNR = 0$dB (left) and for CNR varying with $K = 60$ (right). For clutter subspace estimator derived from the SCM (dark blue), the proposed estimator (light blue), MLE with known texture (black) and MLE with known texture PDF (red). $m = 15$, $r = 3$.

4.3. STAP simulations

STAP [1] is applied to airborne radar in order to detect moving targets. Typically, the radar receiver consists in an array of Q antenna elements processing P pulses in a coherent processing interval ($m = PQ$). In this framework, we assume that the received signal $\mathbf{z}$ is a complex known signal $\mathbf{d}$ corrupted by an additive disturbance $\mathbf{n}$ which follows the general noise model described in Eq. (1) and therefore does not follows the assumption $\lambda_1 = ... = \lambda_r = 1$.

$$\mathbf{z} = \mathbf{d} + \mathbf{n} \tag{21}$$

With a LR clutter, it is well known that a correct sub-optimal filter is [3, 4]:

$$\hat{\mathbf{w}}_{lr} = \hat{\mathbf{\Pi}}_c^\perp \mathbf{d} = \left(\mathbf{I}_m - \hat{\mathbf{\Pi}}_c\right)\mathbf{d} \tag{22}$$

We assume to have K secondary data $\mathbf{z}_i \sim \mathscr{CN}(\mathbf{0}, \mathbf{R}_i)$ which only contain the disturbance to estimate the clutter subspace projector and then process the filtering of $\mathbf{z}$. Of course, the performance of the LR filters will directly rely on the accuracy of the estimation of $\mathbf{\Pi}_c$. To evaluate the performance of a sub-optimal filter, the SINR [1] loss is currenlty used: it is the ratio between the $SINR_{out}$, computed for $\hat{\mathbf{w}}_{lr}$, and $SINR_{max}$ computed for the optimal filter $\mathbf{w} = \mathbf{R}^{-1}\mathbf{d}$. We compare the LR STAP filter built from our new estimator of the subspace projector with the one built from the subspace projector derived from the SCM.

We consider the following STAP configuration. The number Q of sensors is 8 and the number P of coherent pulses is also 8. The center frequency and the bandwidth are respectively equal to $f_0 = 450$ MHz and $B = 4$ MHz. The radar velocity is $100\ m/s$. The inter-element spacing is $d = \frac{c}{2f_0}$ (c is the celerity of light) and the pulse repetition frequency is $f_r = 600$ Hz. The clutter rank is computed from Brennan rule [10] and is equal to $r = 15 \ll 64$, therefore, the low rank assumption is valid. The texture PDF is a Gamma law of shape parameter $\nu = 0.1$ and scale parameter $\frac{1}{\nu}$.

4.3.1. SINR Loss

Figure 3 shows the SINR Loss evolution with respect to K and CNR. We notice that the LR STAP filter built from our estimator still outperforms the LR classical one, which shows the robustness of the approach relative to the hypothesis of equals c_k. Moreover, it reaches performance close to the theoretical optimum. Nevertheless, for high CNR, both LR STAP filters have slightly the same performances. Due to complexity of computation of the term (17), filter based on it is not included in this simulation, which is also justified by the fact that the texture PDF is a priori unknown in this application.

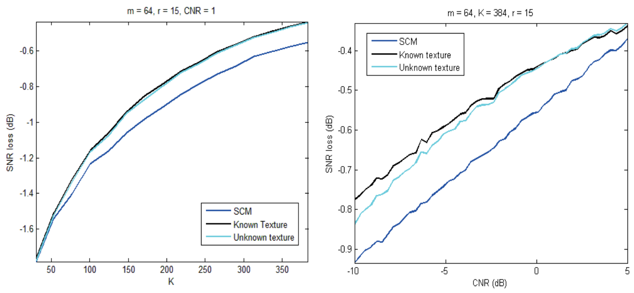

Fig. 3. SINR loss mean on 1000 iterations for K varying with $CNR =$ 0dB (left) and for CNR varying with $K = 384$ (right). For clutter subspace estimator derived from the SCM (dark blue), the proposed estimator (light blue) and MLE with known texture (black). $m = 64$, $r = 15$.

4.3.2. Filter output

For this simulation, a target with a SNR of 0 dB at $\{40$ m/s, $20\deg\}$ is observed in an heterogeneous clutter environment. The total number of secondary data used to estimate $\mathbf{\Pi}_c$ is $K = 30$. The clutter to noise ratio is 0 dB. Figure 4 presents the output of LR STAP filters based on respectively the SCM and estimator from proposition 3.2. It illustrates that the presented estimator allows to ensure a detection with lower false alarm rate than with the estimator derived from the SCM since it provides a better interference rejection.

5. CONCLUSION

In this paper has been introduced a new MLE of the clutter subspace projector in the context of a low-rank SIRV plus white Gaussian noise which outperforms the classical estimator based on the SCM. This estimator does not require prior

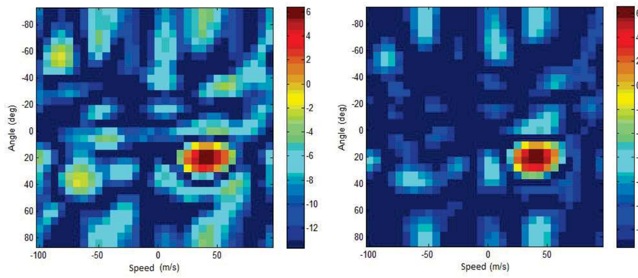

Fig. 4. Filter outputs realized with LR STAP filters built from $\hat{\mathbf{\Pi}}_c$ estimated through SCM (left) and the proposed MLE (right).

knowledge on the texture to be computed. This approach leads, of course, to a loss of performance compared to estimators presented in [9] (with known texture or known texture PDF), but allows to perform an estimation in less restrictive contexts: for example in STAP Radar filtering where no information on the texture is available. Moreover, the presented estimator seems robust to a model variation induced by non equal eigenvalues of the clutter subspace covariance matrix, which is likely in a realistic context.

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