Robust statistical framework for radar change detection applications - Part 1

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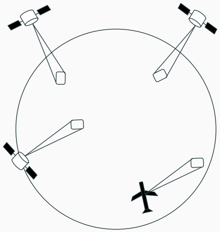
Content

Content

When remote sensing turns into big data

Remote sensing provides various images of the Earth's surface



Huge increase in the number of available acquisitions:

- Sentinel-1: 12 days repeat cycle, since 2014
- TerraSAR-X: 11 days repeat cycle, since 2007
- UAVSAR, ... thousands of flight paths planned

Problem

There is a need for algorithms to process this amount of data automatically!

A focus on change detection problems

Various problems

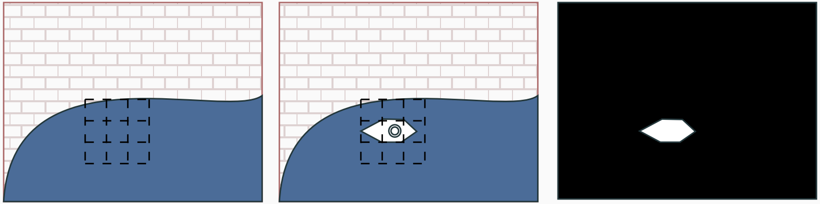
• Target/pattern detection	single snapshot
• Segmentation, classification, clustering, ...	single snapshot, time-series
• **Change Detection**	time-series
• Change estimation (e.g., interferometry)	time-series

Change detection

From a time-series, detect locations where changes occurred over time, e.g.:

- Man-made changes: appearance/disappearance of vehicles/buildings
- Natural disasters: floodings, fires, ...
- Small variations of terrain: glacier displacement, land subsidence

Change detection (CD) problem



$t = 1$ $t = 2$ Ideal CD map

Pixel-level methods

Decide if a change occurred locally (patches) between the observations

Many other approaches in the overview [Hussain et al., 2013]

Change detection application examples

Example 1/2: **activity monitoring**

Figure 1: Terrasar-X images of the Burning-man festival between two dates

Change detection application examples

Example 2/2: **disaster assessment**

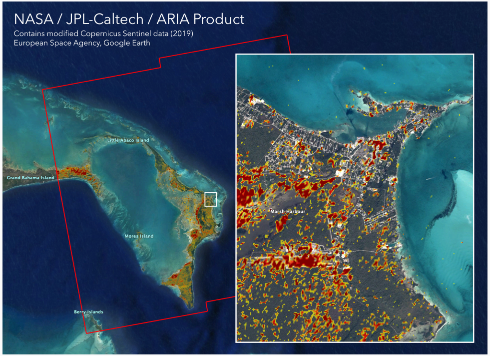
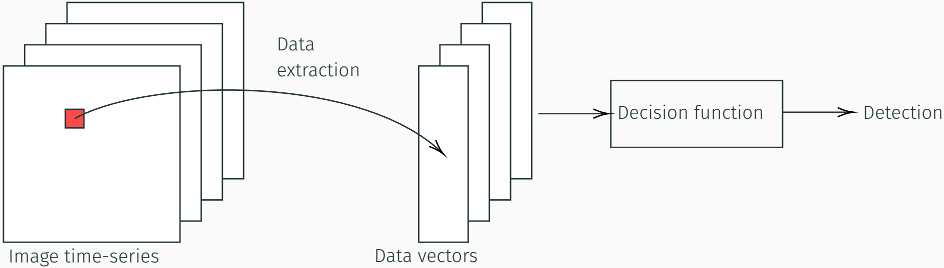


Figure 1: Destruction map of Dorian Hurricane using change detection over Sentinel-1 data

A general framework for pixel-level methods



Two steps:

- **Data extraction**: transform the data to highlight changes we aim to detect
- **Decision function**: compute measure of dissimilarity between data/features

PART 1

- **Sec.2**: Data description
- **Sec.3**: Motivational for statistical and covariance based techniques
- **Sec.4**: Gaussian plug-in detectors (2-step approach)
- **Sec.5**: Gaussian statistical criteria (1 step detection)
- **Sec.6**: Experiments on UAVSAR data

PART 2

- Non-Gaussian models and robust detection
- Detection with structured covariance models

Content

Data: a focus on synthetic aperture radar data

Satellite/airborne remote sensing systems

- RGB optical imaging
- Multispectral/Hyperspectral imaging
- Active sensing: radar, **synthetic aperture radar**

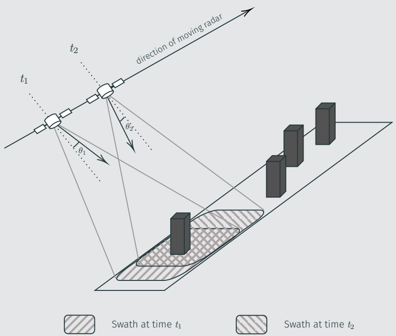
Extensions

Tools from this presentation can be transposed, but require to check assumptions

- Optical images: positive data, non-zero mean, ...
- High dimension issues in hyperspectral imaging

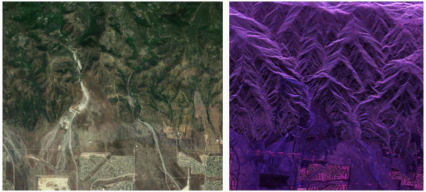
Synthetic aperture radar (SAR)

Principle of SAR



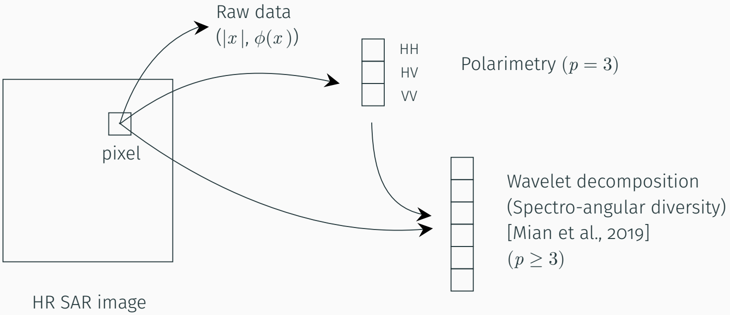
Advantages:

- All weather and illumination conditions (active technology)
- Very high-resolution (sub-meter) imaging
- Cover large areas



Comparison of optical and image

Data extraction (1/3)



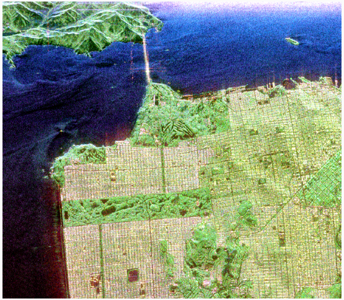
Feature selection

- Leverage **diversity** to improve the detection
- Requires to process **multivariate** pixels

Data extraction (2/3): raw data and polarimetry

Polarimetry $\mathbf{x} = [x_{HH}, x_{HV}, x_{VV}]^T \in \mathbb{C}^3$

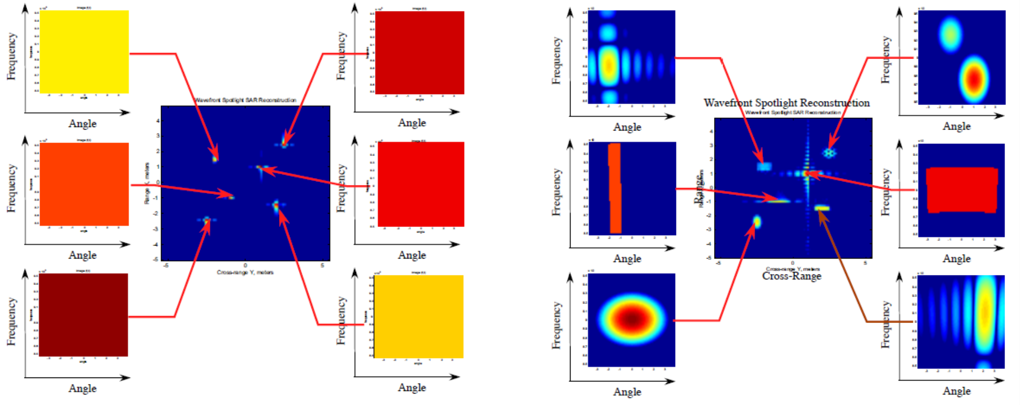
- Pauli decomposition
- Krogager decomposition
- Cameron Decomposition
- H-α decomposition
- An so on...



SF Bay, Pauli basis ($HH - VV$, $\sqrt{2}HV$, $HH + VV$)

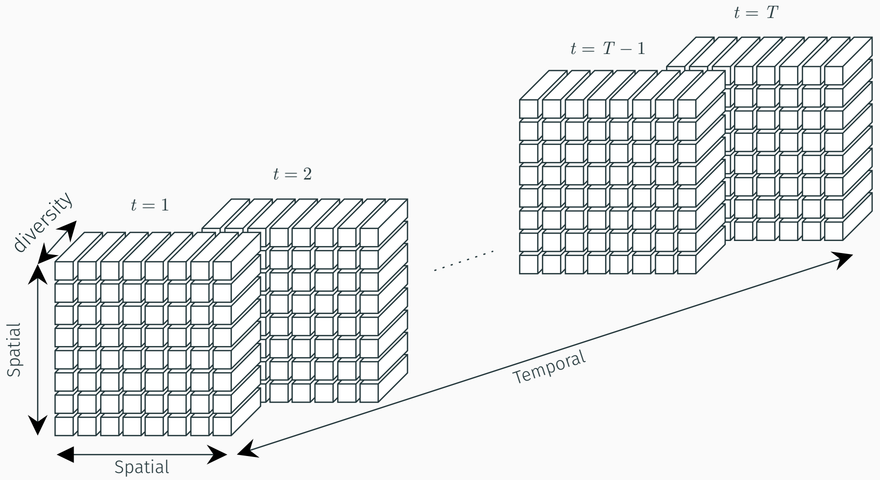
Overview available at
`https://earth.esa.int/documents/653194/656796/Polarimetric_Decompositions.pdf`

Data extraction (3/3): spectro-angular features [Mian et al., 2019]



Wavelet decompositions can retrieve dispersive/anisotropic behavior of the scatterers

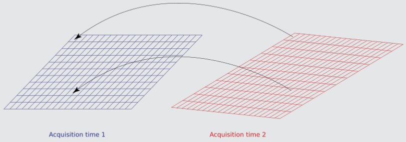
Multivariate SAR Images Time Series



Some issues encountered with SAR images time series (1/2)

Co-registration

- Change detection requires accurate co-registration
- Can be challenging depending on the system (satellite vs plane)



Clutter noise

- Weak Signal to Noise Ratio (SNR)
- Multiplicative noise, Gaussian assumption often not valid

Some issues encountered with SAR images time series (2/2)

Lack of ground truth and labeled data

Obtaining reliable ground truth is extremely complicated and time-consuming

- Comparing with optical data
- Crossing with geographic databases
- Asking local authorities

Our take on this presentation

- Co-registration has been correctly performed
- We will consider **robust models** to handle the noise
- We will focus on the design of **non-supervised** approaches

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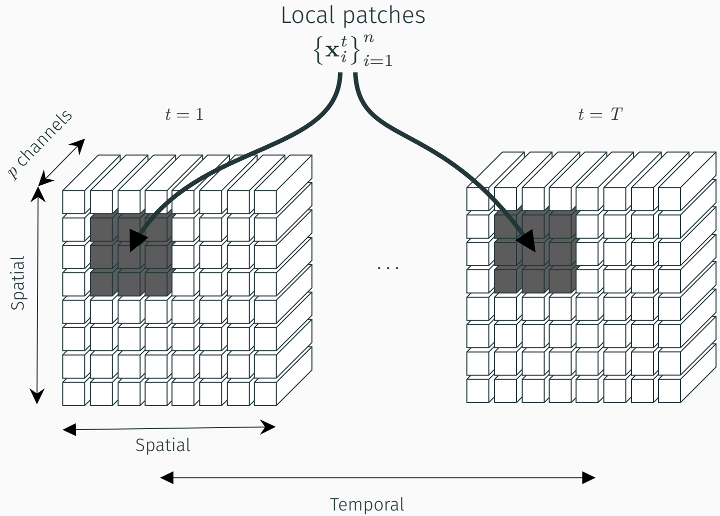
Data: notations



Generic detection process

- **Decision function**: compute measure of dissimilarity between data

$$\begin{aligned} \Lambda : \quad & (\mathbb{C}^{p\times n})^T && \longrightarrow \mathbb{R} \\ & \{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T && \longmapsto \Lambda(\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T) \end{aligned}$$

- **Detection threshold**: decide that a change occurred if

$$\Lambda(\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T) > \lambda$$

- **Ideally**:
  - good trade-off between probability of detection/probability of false alarm (PD/PFA)
  - λ can be set in practice (e.g., CFAR property)

Some classical univariate schemes ($p = 1$, $T = 2$)

Log-ratio [Bazi et al., 2006]

$$\Lambda_{\mathrm{logr}}(\{x_i^1\}_{i=1}^n, \{x_i^2\}_{i=1}^n) = \sum_{i=1}^{n} \log(|x_i^1|/|x_i^2|)$$

Counters the multiplicative nature of the speckle

Coherent change detection (CCD)

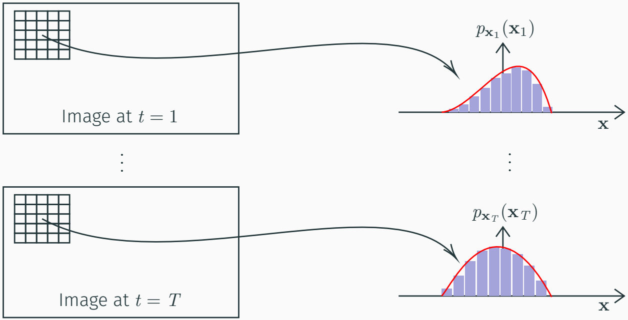
$$\Lambda_{\mathrm{CCD}}(\{x_i^1\}_{i=1}^n, \{x_i^2\}_{i=1}^n) = \frac{2|\sum_{i=1}^n x_i y_i^*|}{\sum_{i=1}^n (|x_i|^2 + |y_i|^2)}$$

Highlight changes in the phase between acquisitions

Multivariate extensions [Novak, 2005, Barber, 2015]

- **Pros**: simple to implement
- **Limitations**: high PFA, univariate, bi-date

Parametric statistical detection (1/4)



- Can handle **multivariate data**
- Can account for **physical modeling** of the data/noise
- Strong **theoretical guarantees** from statistical literature

Parametric statistical detection (2/4)

- Probabilistic model on the observations:

$$\mathbf{x}_i^t \sim p_{\mathbf{x}_i^t}(\mathbf{x}_i^t; \boldsymbol{\theta}_t; \boldsymbol{\Phi}_t), \qquad \begin{cases} \boldsymbol{\theta}_t : \text{Parameters of interest} \\ \boldsymbol{\Phi}_t : \text{Side parameters} \end{cases}$$

- Detect a change in $\boldsymbol{\theta}_t \Leftrightarrow$ binary hypothesis test

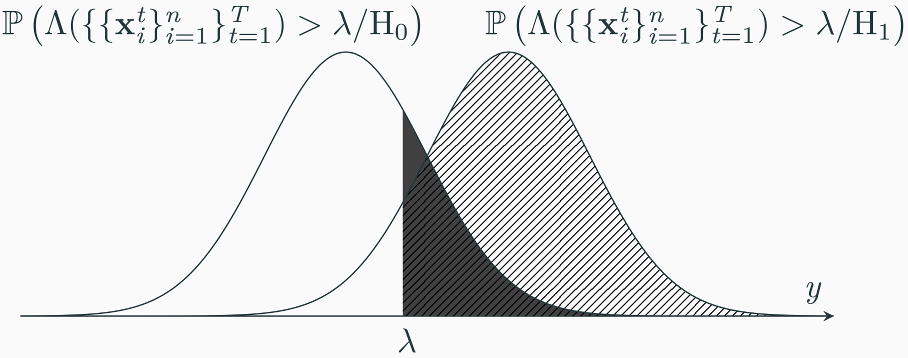
$$\begin{cases} \mathrm{H}_0 : & \boldsymbol{\theta}_1 = \ldots = \boldsymbol{\theta}_T = \boldsymbol{\theta}_0 & \& & \boldsymbol{\Phi}_1 \neq \ldots \neq \boldsymbol{\Phi}_T\,, \\ \mathrm{H}_1 : & \exists (t, t') \in [\![1, T]\!]^2,\, \boldsymbol{\theta}_t \neq \boldsymbol{\theta}_{t'} & \& & \boldsymbol{\Phi}_1 \neq \ldots \neq \boldsymbol{\Phi}_T \end{cases}$$

Problems

- Specify a model and parameters (empirical fit/robustness)
- Find a practical test statistic Λ (decision function)

Parametric statistical detection (3/4)

We expect a **High PD** and **Low PFA** from the detection process (Λ, λ)



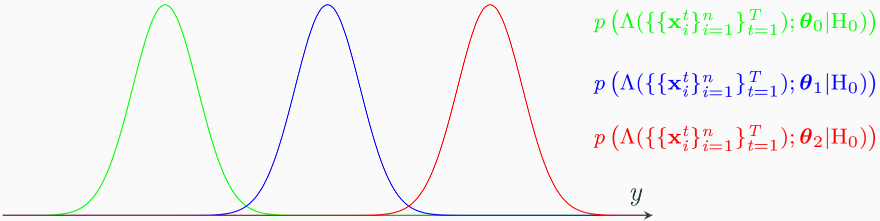
Parametric statistical detection (4/4)

Constant false alarm rate (CFAR)

A statistic Λ is said to be CFAR if $\forall(\boldsymbol{\theta}_0, \boldsymbol{\theta}_1)$, $\forall\lambda$

$$\mathbb{P}\left(\Lambda(\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T; \boldsymbol{\theta}_0|\mathrm{H}_0) > \lambda\right) = \mathbb{P}\left(\Lambda(\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T; \boldsymbol{\theta}_1|\mathrm{H}_0) > \lambda\right)$$

Example of a non CFAR statistic:



Gaussian modeling for statistical CD

Multivariate Gaussian distribution

$\mathbf{x} \sim \mathbb{C}\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $\mathbb{E}\left[\mathbf{x}\right] = \boldsymbol{\mu}$ and $\mathbb{E}\left[\mathbf{x}\mathbf{x}^H\right] = \boldsymbol{\Sigma}$ if it has for p.d.f.

$$p_{\mathbf{x}}(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{\pi^p |\boldsymbol{\Sigma}|^{-1}} \exp\left\{-(\mathbf{x} - \boldsymbol{\mu})^{\mathrm{H}} \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu})\right\}$$

Motivations:

- **Empirical fit** to SAR data (central limit theorem)
  - **zero mean** ($\boldsymbol{\mu} = \mathbf{0}$)
  - **correlation** between channels ($\boldsymbol{\Sigma} \neq \alpha \mathbf{I}$)
- **Practical theoretical results** from statistics and signal processing literature

Gaussian covariance-based CD (1/2)

- **Modeled** by $\mathbf{x}_i^t \sim \mathbb{C}\mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_t)$
- Change in time $\Leftrightarrow$ change in $\mathbf{\Sigma}_t$
- **Omnibus** test ($\boldsymbol{\theta}_t = \mathbf{\Sigma}_t$, $\mathbf{\Phi}_t = \varnothing$)

$$\begin{cases} \mathrm{H}_0: & \mathbf{\Sigma}_1 = \ldots = \mathbf{\Sigma}_T = \mathbf{\Sigma}_0 \\ \mathrm{H}_1: & \forall (t, t') \in [\![1, T]\!]^2, \mathbf{\Sigma}_t \neq \mathbf{\Sigma}_{t'} \end{cases}$$

[Conradsen et al., 2003] "A test statistic in the complex Wishart distribution and its application to change detection in polarimetric SAR data," IEEE Trans. on Geoscience and Remote Sensing, vol. 41, no. 1, pp. 4-19, 2003.

Gaussian covariance-based CD (2/2)

- Problem : $\boldsymbol{\Sigma}_t$ are **unknown** in practice $\Rightarrow$ requires **estimates**
- **Many options** to design Λ!
  - Plug-in detectors (2-step detection)
  - Statistical criteria (1-step detection)
- **Overviews**:

[Ciuonzo et al., 2017] "On Multiple Covariance Equality Testing with Application to SAR Change Detection," IEEE Trans. on Signal Processing, vol. 65, no. 19, pp. 5078-5091, 2017.

[Mian et al., 2020] "An Overview of Covariance-based Change Detection Methodologies in Multivariate SAR Image Time Series", book chapter, Change Detection and Image Time-Series Analysis, Wisley, to appear.

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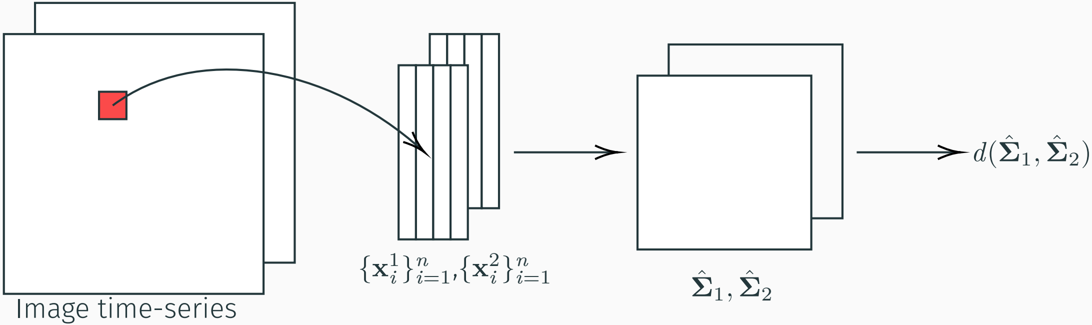
2-step change detection



- **Covariance matrix estimation** (feature extraction)
- Evaluation of a **distance** (feature comparison)

Covariance matrix estimation

Sample covariance matrix (SCM)

Let $\{\mathbf{x}_i\}_{i=1}^n$ following $\mathbf{x} \sim \mathbb{C}\mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$, the ML estimate of $\mathbf{\Sigma}$ is

$$\hat{\mathbf{\Sigma}}_{\text{SCM}} = \frac{1}{n}\sum_{i=1}^{n} \mathbf{x}_i \mathbf{x}_i^H$$

- Simple to implement
- Wishart distributed $\longrightarrow$ well established properties
- Not robust to non-Gaussian/outliers (cf. Part 2)

Distances between covariance matrices

Frobenius	$d_{\mathrm{Fro}}(\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2) = \lVert \boldsymbol{\Sigma}_1 - \boldsymbol{\Sigma}_2 \rVert_F^2$	
Spectral Log	$d_{\mathrm{Log}}(\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2) = \lVert \log(\boldsymbol{\Sigma}_1) - \log(\boldsymbol{\Sigma}_2) \rVert_F^2$	
Hotelling-Lawley	$d_{\mathrm{HTL}}(\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2) = \mathrm{Tr}\left\{\boldsymbol{\Sigma}_1 \boldsymbol{\Sigma}_2^{-1}\right\}$	[Akbari et al., 2016]
KL divergence	$d_{\mathrm{KL}}(\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2) = \mathrm{Tr}\left\{\boldsymbol{\Sigma}_1^{-1} \boldsymbol{\Sigma}_2\right\} + \log\left(\lvert\boldsymbol{\Sigma}_1\rvert / \lvert\boldsymbol{\Sigma}_2\rvert\right)$	[Nascimento et al., 2019]
Wasserstein	$d_{\mathrm{W}}(\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2) = \mathrm{Tr}\left\{\boldsymbol{\Sigma}_1 + \boldsymbol{\Sigma}_2 - 2\left(\boldsymbol{\Sigma}_2^{1/2} \boldsymbol{\Sigma}_1 \boldsymbol{\Sigma}_2^{1/2}\right)^{1/2}\right\}$	[Mian et al., 2020]
Rao	$d_{\mathrm{Rao}}(\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2) = \alpha \sum_{i=1}^p \log^2 \lambda_i + \beta \left(\sum_{i=1}^p \log \lambda_i\right)^2$, $\{\lambda_i\}_{i=1}^p = \mathrm{eig}(\boldsymbol{\Sigma}_1^{-1} \boldsymbol{\Sigma}_2)$	[Ratha et al., 2017]

2-step change detection

Gaussian plug-in detectors

$$\Lambda(\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T) = d(\hat{\mathbf{\Sigma}}_{\mathrm{SCM}}^1, \hat{\mathbf{\Sigma}}_{\mathrm{SCM}}^2)$$

Advantages

- Practical anf flexible
- Estimates: many options
- Distance: various invariances
- Wishart characterization

Limitations

- $T = 2$
- CFAR: case by case study
- 2-step $\rightarrow$ "suboptimal"?

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1-step change detection

- **Model** $\mathbf{x}_i^t \sim \mathbb{C}\mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_t)$
- **Binary hypothesis test**

$$\begin{cases} \mathrm{H}_0: & \mathbf{\Sigma}_1 = \ldots = \mathbf{\Sigma}_T = \mathbf{\Sigma}_0 \\ \mathrm{H}_1: & \forall (t, t') \in [\![1, T]\!]^2, \mathbf{\Sigma}_t \neq \mathbf{\Sigma}_{t'} \end{cases}$$

$\Rightarrow$ Derive Λ following well-established decision statistics

- Many criteria exist, but most of them are equivalent to either GLRT, t_1, or Wald for the considered problem [Ciuonzo et al., 2017]

Generalized likelihood ratio test (GLRT) (1/3)

GLRT **[Kay and Gabriel, 2003]**

Given the data $\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T$, model $p_{\mathbf{x}}$ and test parameters $\{\boldsymbol{\theta}, \boldsymbol{\Phi}\}$, the GLRT is

$$\hat{\Lambda}_{\mathrm{GLRT}} = \frac{\max\limits_{\{\boldsymbol{\theta}_t, \boldsymbol{\Phi}_t\}_{t=1}^T} p_{\mathbf{x}}\left(\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T \,;\, \{\boldsymbol{\theta}_t, \boldsymbol{\Phi}_t\}_{t=1}^T \mid \mathrm{H}_1\right)}{\max\limits_{\boldsymbol{\theta}_0, \{\boldsymbol{\Phi}_t\}_{t=1}^T} p_{\mathbf{x}}\left(\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T \,;\, \boldsymbol{\theta}_0, \{\boldsymbol{\Phi}_t\}_{t=1}^T \mid \mathrm{H}_0\right)} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \lambda.$$

Generalized likelihood ratio test (GLRT) (2/3)

GLRT for covariance change detection

Assuming that the data $\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T$ follows $\mathbf{x}_i^t \sim \mathbb{C}\mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_t)$, the GLRT for the test

$$\begin{cases} \mathrm{H}_0: & \mathbf{\Sigma}_1 = \ldots = \mathbf{\Sigma}_T = \mathbf{\Sigma}_0 \\ \mathrm{H}_1: & \forall (t, t') \in [\![1, T]\!]^2, \, \mathbf{\Sigma}_t \neq \mathbf{\Sigma}_{t'} \end{cases}$$

is

$$\hat{\Lambda}_{\mathrm{G}} = \frac{\left|\hat{\mathbf{\Sigma}}_{\mathrm{SCM}}^0\right|^{nT}}{\prod_{t=1}^{T} \left|\hat{\mathbf{\Sigma}}_{\mathrm{SCM}}^t\right|^{n}},$$

where $\hat{\mathbf{\Sigma}}_{\mathrm{SCM}}^t$ is the SCM of $\{\mathbf{x}_i^t\}_{i=1}^n$ and $\hat{\mathbf{\Sigma}}_{\mathrm{SCM}}^0$ is the SCM of $\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T$

Generalized likelihood ratio test (GLRT) (3/3)

Univariate case: Incoherent change detection (ICD) **[Mian et al., 2017]**

For $p = 1,\ T = 2$, the GLRT reduces to

$$\Lambda_{\mathrm{ICD}}(\{x_i^1\}_{i=1}^n, \{x_i^2\}_{i=1}^n) = \frac{\left(\sum_{i=1}^n |x_i^1|^2 + \sum_{i=1}^n |x_i^2|^2\right)^2}{\sum_{i=1}^n |x_i^1|^2 \sum_{i=1}^n |x_i^2|^2}$$

Generalized likelihood ratio test on the variance

Terrell (gradient) statistic (1/2)

Terrell statistic **[Radhakrishna Rao, 1948, Terrell, 2002]**

Given the data $\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T$, model $p_{\mathbf{x}}$ and test parameters $\boldsymbol{\theta}$, the t_1 is

$$\Lambda_{t_1} = \left.\frac{\partial \log p_{\mathbf{x}}\left(\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T\,;\,\{\boldsymbol{\theta}_t, \boldsymbol{\Phi}_t\}_{t=1}^T \mid \mathrm{H}_1\right)}{\partial \boldsymbol{\theta}^T}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}_0} (\hat{\boldsymbol{\theta}}_1 - \hat{\boldsymbol{\theta}}_0)$$

Terrell (gradient) statistic (2/2)

t_1 statistic for covariance change detection

Assuming that the data $\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T$ follows $\mathbf{x}_i^t \sim \mathbb{C}\mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_t)$, the t_1 for the test

$$\begin{cases} \mathrm{H}_0: & \mathbf{\Sigma}_1 = \ldots = \mathbf{\Sigma}_T = \mathbf{\Sigma}_0 \\ \mathrm{H}_1: & \forall (t, t') \in [\![1, T]\!]^2, \ \mathbf{\Sigma}_t \neq \mathbf{\Sigma}_{t'} \end{cases}$$

is

$$\hat{\Lambda}_{\mathrm{t}_1} = \frac{1}{T} \sum_{t=1}^{T} \mathrm{Tr}\left[\left(\left(\hat{\Sigma}_{\mathrm{SCM}}^{0} \right)^{-1} \hat{\Sigma}_{\mathrm{SCM}}^{t} \right)^2 \right]$$

where $\hat{\mathbf{\Sigma}}_{\mathrm{SCM}}^t$ is the SCM of $\{\mathbf{x}_i^t\}_{i=1}^n$ and $\hat{\mathbf{\Sigma}}_{\mathrm{SCM}}^0$ is the SCM of $\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T$

Wald statistic (1/2)

Wald statistic **[Wald, 1943]**

Given the data $\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T$, model $p_{\mathbf{x}}$ and test parameters $\boldsymbol{\theta}$, the Wald statistic is

$$\Lambda_{\text{Wald}} = (\hat{\boldsymbol{\theta}}_1 - \boldsymbol{\theta}_0)^H \left(\left[\mathbf{I}^{-1}(\hat{\boldsymbol{\theta}}_1) \right]_{\boldsymbol{\theta}} \right)^{-1} (\hat{\boldsymbol{\theta}}_1 - \boldsymbol{\theta}_0) \,,$$

where $\mathbf{I}(\boldsymbol{\theta})$ is the Fisher information matrix of the estimation problem under the H_1

Wald statistic (2/2)

Wald statistic for covariance change detection

Assuming that the data $\{\{\mathbf{x}_i^t\}_{i=1}^n\}_{t=1}^T$ follows $\mathbf{x}_i^t \sim \mathbb{C}\mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_t)$, the Wald statistic for

$$\begin{cases} \mathrm{H}_0: & \boldsymbol{\Sigma}_1 = \ldots = \boldsymbol{\Sigma}_T = \boldsymbol{\Sigma}_0 \\ \mathrm{H}_1: & \forall (t, t') \in [\![1, T]\!]^2, \, \boldsymbol{\Sigma}_t \neq \boldsymbol{\Sigma}_{t'} \end{cases}$$

is

$$\hat{\Lambda}_{\mathrm{Wald}} = n \sum_{t=2}^{T} \mathrm{Tr}\left[\left(\mathbf{I} - \hat{\boldsymbol{\Sigma}}_{\mathrm{SCM}}^{1} (\hat{\boldsymbol{\Sigma}}_{\mathrm{SCM}}^{t})^{-1}\right)^2\right] - q\left(n \sum_{t=1}^{T} (\hat{\boldsymbol{\Sigma}}_{\mathrm{SCM}}^{t})^{-T} \otimes (\hat{\boldsymbol{\Sigma}}_{\mathrm{SCM}}^{t})^{-1}, \mathrm{vec}\left(\sum_{t=2}^{T} \boldsymbol{\Upsilon}_t\right)\right)$$

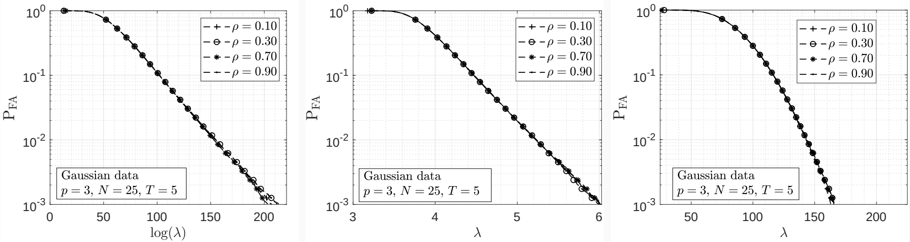
with $\boldsymbol{\Upsilon}_t = N\left((\hat{\boldsymbol{\Sigma}}_{\mathrm{SCM}}^{t})^{-1} - (\hat{\boldsymbol{\Sigma}}_{\mathrm{SCM}}^{t})^{-1} \hat{\boldsymbol{\Sigma}}_{\mathrm{SCM}}^{1} (\hat{\boldsymbol{\Sigma}}_{\mathrm{SCM}}^{t})^{-1}\right)$ and $q(\mathbf{x}, \boldsymbol{\Sigma}) = \mathbf{x}^{\mathrm{H}} \boldsymbol{\Sigma}^{-1} \mathbf{x}$

Some properties of the statistics

CFARness properties

The GLRT, t_1 and Wald statistics are CFAR w.r.t. the covariance parameter

In simulation: $\mathbf{x}_k^t \sim \mathbb{C}\mathcal{N}\left(\mathbf{0}_p, (\rho^{|i-j|})_{ij}\right)$.



Content

Data set description

UAVSAR (courtesy of NASA/JPLCaltech, https://uavsar.jpl.nasa.gov)

Dataset	Resolution	Scene	p	T	Size	Coordinates (top-left px)
UAVSAR SanAnd_26524_03 Segment 4 April 23, 2009 - May 15, 2011	Rg: 1.67m Az: 0.6m	Scene 1	3	2	2360×600 px	[Rg, Az] = [2891, 28891]
		Scene 2	3	2	2300×600 px	[Rg, Az] = [3236,25601]
Snjoaq_14511		Scene 3	3	17	2300×600 px	[Rg, Az] = [3236,25601]

Source: https://github.com/ammarmian

Data set description

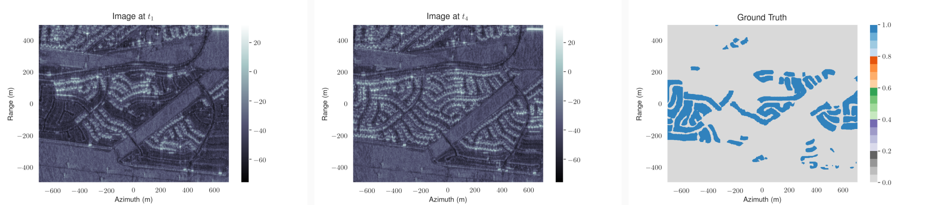


Figure 2: UAVSAR Scene 1, ground truth from [Ratha et al., 2017, Nascimento et al., 2019]

Data set description

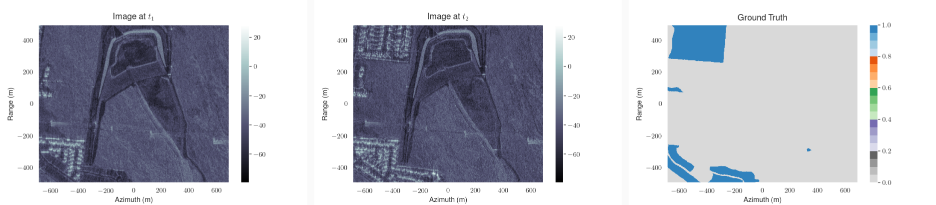


Figure 3: UAVSAR Scene 2, ground truth from [Ratha et al., 2017, Nascimento et al., 2019]

Data set description

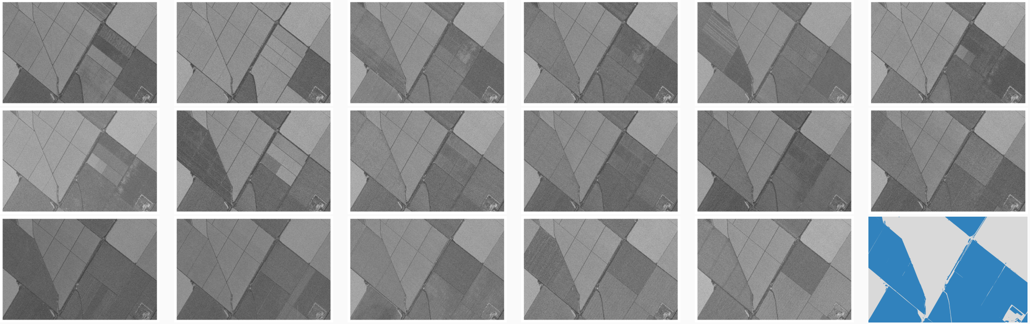


Figure 4: UAVSAR Scene 3, ground truth from [Mian et al., 2020]

Compared detectors

- Plug-in detectors using SCMs ($T = 2$)
  - Λ_{HTL} Hotelling-Lawley divergence
  - Λ_{KL} KL divergence
  - $\Lambda_{\mathcal{RG}}$ Riemannian distance (Rao distance with $\alpha = 1, \beta = 0$)
  - $\Lambda_{\mathcal{WG}}$ Wasserstein distance
- Gaussian statistical criteria ($T \geq 2$)
  - Λ_{G} GLRT
  - Λ_{t_1} Terrell statistic
  - Λ_{Wald} Wald statistic

Results scene 1-2 ($T = 2$)

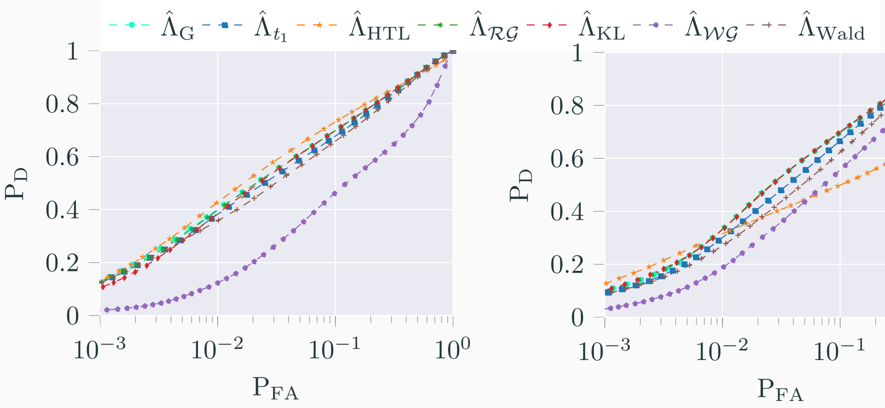


Figure 5: ROC plots using a 5×5 local window for the scenes 1 and 2.

Results scene 3 ($T > 2$)

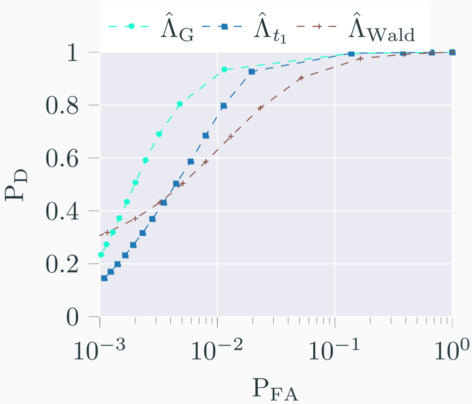


Figure 6: ROC plots using a 5×5 local window for the scene 3.

Content

Conclusion

- Change detection in multivariate image time series
- Statistical framework
  - assumes a distribution
  - Choose parameters of distribution
  - derive test statistics (decision function Λ)
- Gaussian framework with covariance matrix (good local feature to assess for changes with multivariate data)
  - Plug-in detectors (matrix distance) using the SCM
  - Statistical criteria: CFAR property

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